Designing Approximately Optimal Search on Matching Platforms

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Online Matching Platforms

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tinder Uber upwork™ formerly oDesk DOORDASH airbnb

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Match Recommendations as Search

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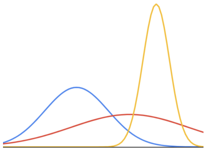
The platform's search design should be:

- informed by data, but also account for uncertainty
- robust to user adaptation in response to the design

Our Model: Data-driven Search Design

Model:

- Platform has **distributional knowledge** of preferences via **agent types**



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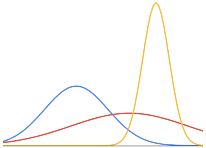

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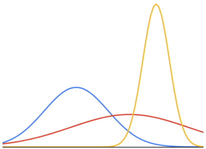

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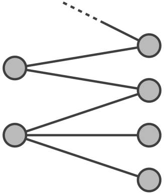

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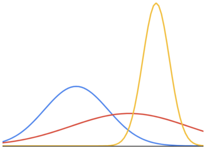

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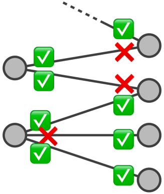

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Challenges:

- Must handle **uncertainty** about agent preferences
- Agents can strategically **hold out** for better matches
    - Leads to congestion and cannibalization in the market

Our Model: Algorithmic Results and Takeaways

Develop efficient 4-approximation algorithm for the search design problem:

- algorithm attains 1/4 of the first-best welfare
- (approximation necessary—even 1.01-approximation is NP-hard)

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Takeaway: Through careful search design, platform can induce equilibrium outcome with almost (socially) optimal welfare

Related Work

Search and matching. Long history of search in the context of matching markets: Burdett and Coles (*QJE*, 1997), Shimer and Smith (*Econometrica*, 2000), Adachi (*JET*, 2003), ... But these works assume *random meeting* of agents and do not consider the problem of designing search

Online matching platforms. Rios et al. (EC 2021) empirically study assortment optimization for online dating; Kanoria and Saban (*Manag. Sci.*, 2021) study online marketplace design with search frictions; Shi (EC 2020) studies matchmaking strategies on two-sided matching platforms; Halaburda et al. (*Manag. Sci.*, 2018) model the effect of restricting choice; Ashlagi et al. (WINE 2020) study two-sided assortment optimization with a multinomial logit choice function; ...

Model

Agents, Types, and Dynamics

Continuum model—agents have infinitesimal mass

Each agent has one of finitely many **types** $\boldsymbol{\theta} \in \boldsymbol{\Theta}$

Types divided into two sides: $\Theta = \mathcal{M} \sqcup \mathcal{W}$

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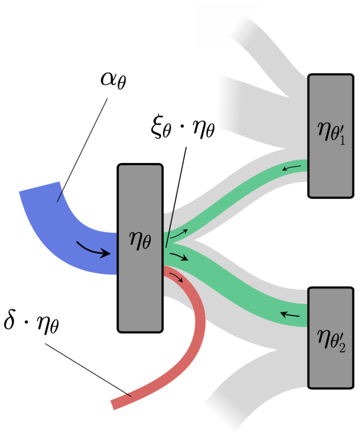
Agents of type θ enter at exogenous **arrival rate $\boldsymbol{\alpha_\theta}$**

Departure is endogenous—agents either leave:

(a) **matched**—by entering into a mutually agreed upon match, or

(b) **unmatched**—by experiencing a "life event" at an exogenous (per-agent) rate of $\boldsymbol{\delta\, dt}$



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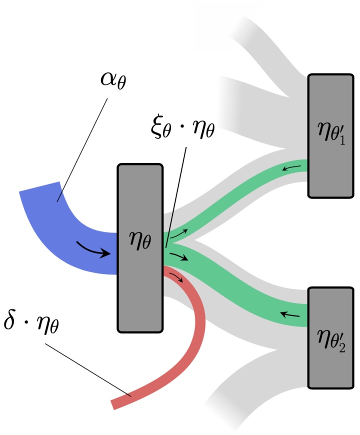
Each type has a stationary population mass η_θ from balancing inflows against outflows



Preferences and Utilities

Agents have random, symmmetric, cardinal utilities determined by their types:

- each pair of types $(m, w) \in \mathcal{M} \times \mathcal{W}$ has continuous utility distribution F_{mw}
- each pair of type m and w agents has shared utility for matching $\sim F_{mw}$
- utility for each pair of agents is drawn independently

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Conversely, assume platform knows distributions F_{mw} for all $(m, w) \in \mathcal{M} \times \mathcal{W}$

- Platform's data $\implies$ distributional knowledge

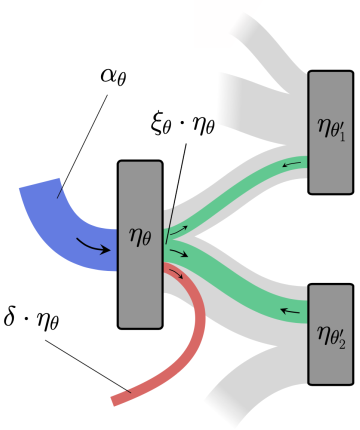
Directed Search

Platform sets rates at which pairs of types meet

- Type θ agents meet type θ' agents according to Poisson process of **rate $\lambda_\theta(\theta')$**

Upon pair of type m and w agents meeting, each sees utility $u \sim F_{mw}$, then **accepts**/**rejects** match

- Match (and leave) with utility u iff both accept



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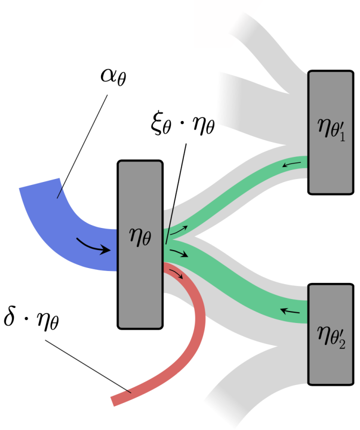
Rates $\lambda_\theta(\theta')$ subject to feasibility constraints:

(a) **capacity** constraint: ≤ 1 meeting / unit time

$$\sum_{\theta'} \lambda_\theta(\theta') \leq 1$$

(b) **flow** constraint: equal #s of any two types meet

$$\eta_m \lambda_m(w) = \eta_w \lambda_w(m)$$



The Design Problem

The Platform's Design Problem

Platform sets rates subject to feasibility constraints (capacity + flow)

Feasible choice of rates $\implies$ game among agents

- **Structural result:** unique Nash equilibrium always exists!

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Platform's objective: maximize social welfare in stationary equilibrium

The Optimization Program

The resulting platform optimization problem:

$$\max_{\lambda_\theta, \tau_\theta, \xi_\theta, \eta_\theta} \quad 2 \cdot \sum_{m \in \mathcal{M}} \sum_{w \in \mathcal{W}} \left(\eta_m \lambda_m(w) \int_{\max(\tau_m, \tau_w)} u \, dF_{mw} \right)$$

such that

$$\alpha_\theta = (\delta + \xi_\theta)\eta_\theta \qquad \text{(stationarity)}$$

$$\eta_m \lambda_m(w) = \eta_w \lambda_w(m) \qquad \text{(flow)}$$

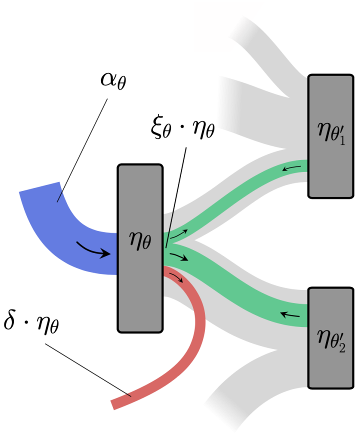
$$1 \geq \sum_{\theta'} \lambda_\theta(\theta') \qquad \text{(capacity)}$$

$$\xi_\theta = \sum_{\theta'} \left(\lambda_\theta(\theta') \int_{\max(\tau_\theta, \tau_{\theta'})}^{\infty} dF_{\theta\theta'} \right) \qquad (\xi_\theta \text{ defn})$$

$$\tau_\theta = \frac{1}{\delta + \xi_\theta} \sum_{\theta'} \left(\lambda_\theta(\theta') \int_{\max(\tau_\theta, \tau_{\theta'})}^{\infty} u \, dF_{\theta\theta'} \right)$$

(agents' best response)

$$\lambda_m(w), \lambda_w(m) \geq 0. \qquad \text{(nonnegativity of rates)}$$



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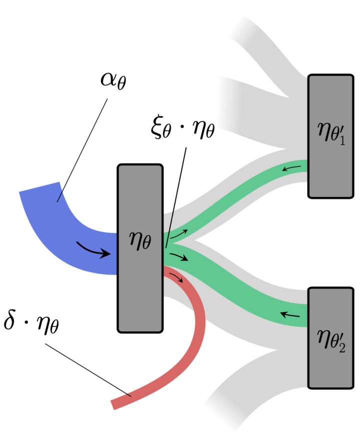

Approximation Algorithm

Algorithmic Result

Theorem

The platform can efficiently find a 4-approximately optimal search design.

Welfare of our construction **4-approximates** welfare of the **first-best** outcome

- First best = "planned market" where platform can choose agent strategies

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- First best = "planned market" where platform can choose agent strategies

Algorithm sketch: Construct approximately optimal search design in two phases:

1. **Solve for the first best.** Exactly computing the first-best outcome turns out to be a computationally tractable problem
2. **Approximate first best via star-shaped submarkets.** Divide market into submarkets based on first best; then reintroduce incentives

Phase I: Finding the First Best

1. Relax agent best response constraint
2. Rewrite optimization problem in terms of a “cutoff” for each pair of types
3. Messy initial optimization problem reduces to a generalized assignment problem!

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   - Optimal solution has the structure of a **tree over agent types**

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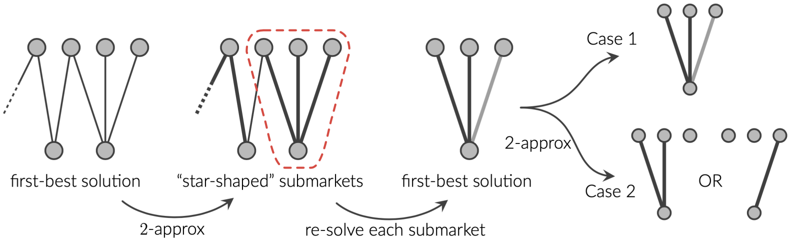
Phase II: Approximation via Star-shaped Submarkets



1. 2-approximate (tree-shaped) first-best solution with *star*-shaped submarkets
   - Akin to Lenstra et al. (*Math. Program.*, 1990), Banerjee et al. (WWW 2017)
2. Re-solve for the first-best solution in each submarket
3. Adjust the new first-best in each submarket into a stationary equilibrium outcome while losing at most a 2-factor in welfare

Conclusion

Summary + Takeaways

In this work, we:

- Connected match recommendations to search design for matching markets
- Investigated the challenges of making match recommendations when facing incomplete knowledge of preferences and strategic agents
- Developed an efficient algorithm to find an approximately optimal search design for general preference distributions

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Takeaway: Through careful search design—which can involve limiting agents' choice, the platform can induce equilibrium outcome with almost (socially) optimal welfare